Data Analysis
User's Guide

Chapter II:
Single Particle Imaging

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2.0 Single Particle Imaging

2.1 Theory

2.1.1 Size Determination

The optical array probe (OAP) was one of the first instruments used to measure precipitation-sized cloud particles (Knollenberg, 1972). Since its earliest implementation, it has been refined so that it is now being used to measure cloud particles smaller than 25 μm. As shown in Fig. 2.1, the OAP uses a collimated laser beam to illuminate a linear array of photodiodes. As a particle passes through this beam, a shadow image is cast on the diodes. This image is captured by recording the intensity level of light during the passage of the particle and a count of the total number of occulted diodes represents the particle size. The diodes at each end of the array act as a mechanism for flagging those particles which do not pass entirely within the bounds of the linear array and would otherwise be undersized.

The actual implementation of image capture in the DMT cloud imaging probe (CIP) is accomplished by measuring the intensity levels of the 64 diodes in the linear array at a frequency rate, f,

\[ f = \frac{V}{\text{Res}} \]  

(2.1)

where

\[ V \] = velocity of particles passing through the beam
\[ \text{Res} \] = size resolution of the CIP, typically 15, 25, 50, or 100 μm

Clocking at this rate assures that images are captured at full resolution, i.e. that undersampling does not lose information and produce truncated images or that they are not oversampled, producing elongated images. This is discussed in greater detail in section 2.2, uncertainties and errors. The sample that is taken at each time step represents a data slice and the sample rate is often referred to as the “slice rate”.

The correct clocking is maintained by measuring the airspeed and calculating the correct frequency with (2.1). The CIP, whether operated by itself or as part of the CAPS, has a pitot static tube for measuring the airspeed at the point of measurement and calculating the true air speed (TAS). The TAS can also be measured independently and transmitted by the data system to the CIP where it is converted to a clock rate. The user chooses which of the two methods to use.

The intensity of light on the diode array is converted to an electrical current and conditioned with the electronics before being digitized with an analog to digital (A/D)
converter. As shown in Fig. 2.2 (from Korolev et al, 1998), the darkness of a particle’s shadow image depends on how close to the center of focus it is imaged. In Fig. 2.2, the left column shows the relative, intensity cross section of a shadow image. A level of unity is for no obscuration (or shadowing) and a level of zero is maximum shadowing. Moving from the top to the bottom of the figure, the particle is imaged at increasingly greater distances from the center of focus. We see that two changes occur in the intensity cross section of the shadow: 1) the shadow becomes less intense, i.e. less dark and 2) the width of the image increases, i.e. the image is no longer an accurate representation of the object being imaged.

The other three columns show simulations of how the object would look imaged on the diodes but using different intensity thresholds to determine the ‘on’ or ‘off’ state of the diode. For example, the column labeled 25% shows the most detail of a particle’s image since it uses gray levels of 25%, 50% and 75%. This rendition of a particle’s image shows how the image becomes lighter and less distinct as the particle is imaged further away from the center of focus. Similar changes are seen using thresholds of 50% and 75% but now we see that instead of becoming lighter in shading, the image is completely blank in some regions, particularly in the middle.

In the earliest implementation of the OAP, images were recorded in black and white, i.e. either a diode was determined as ‘ON’ (black) or ‘OFF’ (white). Through experimentation it was found that the diameter of the image of a water drop would be within 10% of the actual drop diameter when the darkness of the image was at least 50% of total obscuration (Knollenberg, 1972). In terms of how the image recording is actually
implemented in practice, when the intensity of laser light on the diode array decreases by more than 50% due to occultation by a particle, the resulting image size will be accurate to within 10%.
Figure 2.2
The light intensity on each of the photodiodes of the CIP-mudoscale is monitored as a voltage level and at each clock pulse, described above, this voltage is compared to a running average that, with no particles in the beam, may change slightly as a result of alignment. When at least one of the photodiode’s voltage level decreases by 50% from the average, the ON/OFF state is recorded from all 64 diodes since the 50% decrease indicate the presence of a particle in the beam and within an acceptable distance from the center of focus.

The ON/OFF state of the diodes is continuously recorded at the rate determined by (2.1) until the light intensity returns to its ambient level on all 64 diodes. The recorded images are subsequently recreated by display the matrix of ON/OFF pixels as shown in Fig. 2.3.

![Figure 2.3](image1)

The implementation of image capture in the CIP-Gray is identical to that in the CIP-Mono with respect to how the sample rate is determined and the light intensity monitored. The difference is that the CIP-Gray can be programmed to record more that just the ON/OFF state of each diode. The user decides what minimum percentage decrease in light intensity will denote the presence of a particle. Once detected, the light level of each photodiode is recorded as a percentage decrease from the ambient. This means that rather than just ‘black and white’, the images are reconstructed as levels of ‘gray’.

Figure 2.4 shows an example of ice crystal images that were recorded with a CIP-grayscale probe with 15 μm resolution and with three levels of gray representing 25%, 50% and 70% decreases in light intensity on the photodiodes.

![Figure 2.4](image2)
The principal advantage of recording multiple levels rather than just a single 50% decrease is that it allows a more precise determination of the particle depth of field, as is discussed in the next section.

2.1.2 Sample Volume Determination

The particle measured by the CIP are usually characterized by the same properties as in the CAS, CDP and Fog Monitor, i.e. the total concentration, mean and median diameters, effective radius, the liquid water content and extinction. The definition of these parameters is the same as given in Chapter one, section 1.3.2 The difference is in how the sample volumes are determined.

2.1.2.1 Effective Array Width

Unlike the light scattering probes, whose sample volumes do not vary with particle size, the sample volume of an OAP is a strong function of the particle size. Instead of an effective beam diameter (EBD), defined for the light scattering probes, the CIP has an effective array width (EAW), that decreases or increases with the size of the particle measured, depending on whether the user wishes to accept or reject particles that are partially outside of the field of view. As shown in Fig. 2.5 (from Heymsfield and Parrish, 1978), a particle can be totally within the diode array (top panel), partially in with one of the end elements shadowed (middle panel) or partially in with both end elements obscured (bottom panel).

![Figure 2.5](image-url)
When the image is completely within the diode array, this is referred to as “all-in”. For particles that are symmetrical in cross section, their center can be located, even when covering one or both end elements. In this case, the image is referred to as “center-in”, when the center is located within the diode array. In the case for “all-in” or “center-in”, the EAW is calculated differently (Heymsfield and Parrish, 1978).

The “all-in” definition for the EAW is

\[
EAW = \text{Res}(N - X - 1)
\]  

(2.2)

Where the resolution, \( \text{Res} \), is in units of centimeter, \( N \) is the number of photodiodes and \( X \) is the number of diodes shadowed by a particle. An image that covers either or both of the end elements will be rejected. Equation (2.2) takes into account the increasing probability as a function of size that a particle will shadow an end diode and be rejected.

To understand this formulation, a simple example will demonstrate the reason behind this. Let’s say we have a diode array that is 6 elements wide, and number them diodes 1-6, with 1 and 6 being the end diodes. A particle that is one element wide can have its center pass over the center diodes 2-5 and it will be accepted. However, this same particle will not be accepted within the full width of diodes 2-5 because, if its center passes along the edge of either of diodes 2 or 5, it will cover diodes 1 or 6 by 50%. Hence, 1/2 a diode must be subtracted from either end of the array, i.e. one diode in total, to take this into account. This leads us to the conclusion that in this example, a particle of single diode width has an effective array width of 4 diodes and fits the formulation of (2.2). Using this same reasoning, we see that a particle that is 2 diodes wide will shadow end diodes if it is within 1.5 diodes of either end and this now decreases the effective array width for this particle by 1. This also is described properly by equation 2.2.

The EAW for quasi-symmetrical particles whose centers can be located within the diode array, using the technique described by Heymsfield and Parrish (1978), is defined as

\[
EAW = 64*\text{Res}
\]  

(2.3)

It is clearly seen here that the EAW using “center-in” is larger than for “all-in”, but it requires that particles are symmetrical so that the location of their center can be accurately determined.

### 2.1.2.2 Depth of Field

The DOF of an OAP is always increasing function of size, limited only by the distance between the arms of the instrument. By strict definition, what we call here the DOF is not a depth-of-field as used in optical design. It is being used here strictly to define that...
length of the beam where a particle will shadow a photodiode by at least 50%. A particle that is exactly at the focal plane of the OAP optics will form a shadow on the diode array that will occult the detector by 100%, as seen in Fig. 2.2 in the top most row of images. As a particle passes through the beam a further distance from this point, however, diffraction effects cause the particle shadow to increase in size and decrease in intensity. This is a function of the particle diameter and, as described previously and the probe electronics have been designed to reject any particle whose shadow cannot cause a decrease in light intensity on the detector array of greater than a prescribed percentage. Knollenberg (1972) found that the relationship between the acceptable “DOF” and particle diameter, when using 50% decrease as the threshold for shadowing, was

\[
\text{DOF} = \frac{\pm 3R^2}{\lambda}
\]

(2.4)

where

\( R \) = radius of the particle in centimeters
\( \lambda \) = the laser wavelength in centimeters

The DOF of the instruments is limited by the distance between probe arm tips. The maximum depth of field for the CIP is 100 mm, which corresponds to a particle radius of 106 \( \mu \)m. Therefore, when calculating the DOF, the value must be truncated at 100 mm.

Figure 2.6 illustrates the sample areas as a function of diameter for probe resolutions of 15 \( \mu \)m (CIP-Gray), 25 \( \mu \)m (CIP-Mono), 50 \( \mu \)m (MPS) and 100 \( \mu \)m (PIP).
2.2 Uncertainties and Errors

2.2.1 Clocking Errors

The synchronization of imaging recording with air speed was discussed in section 2.1 where the importance of correctly sampling the photodiode voltages was introduced. As shown in Fig. 2.7, with respect to the velocity of a water drop, if the sample rate is too fast, the recorded image is elongated because it is oversampled (left image) and if the sample rate is too slow, the image is truncated (right image). The center image in Fig. 2.7 shows a drop image that has the sample rate correctly synchronized with the particle velocity.
An examination of image shape in all water clouds is a useful diagnostic tool to verify that the instrument is correctly sampling the particles. There are two main reasons that the OAP may under or oversample the cloud particles: 1) the airspeed is being incorrectly measured or computed or 2) the velocity of the particles is different than the ambient air speed.

The CIP-mono. CIP-grayscale and PIP have a pitot-static tube that measures the static pressure, $P_s$, the dynamic pressure, $Q_c$, and a sensor for measuring the ambient temperature, $T_m$. With these three parameters the air velocity is derived near the sample volume of these instruments. The equation to derive air speed is

$$\text{TAS} = M \times 20.06 \times T_a^{0.5} \quad (2.5)$$

$$M = \text{Mach number} = \{2(C_v/R)[(Q_c/P_s +1)^{R/Cp} -1]\}^{0.5}$$

$$T_a = T_m / [1 + r \, M^2 \, (\Gamma - 1)/2]$$

$$T_m = \text{Measured (recovered) Ambient Temperature} + 273.16$$

$$Q_c = \text{Dynamic (Pitot) Pressure (house keeping value number 3)}$$

$$P_s = \text{Static Pressure (house keeping value number 4)}$$

$$C_p = \text{specific heat at constant pressure: } 0.24 \text{ cal g}^{-1} \text{ K}^{-1}$$

$$C_v = \text{specific heat at constant volume: } 0.171 \text{ cal g}^{-1} \text{ K}^{-1}$$

$$R = \text{gas constant for dry air: } 6.8557 \times 10^{-2} \text{ cal g}^{-1} \text{ K}^{-1}$$

$$\Gamma = C_p / C_v = 1.4$$

$$r = \text{Recovery coefficient of the temperature sensor.}$$

Figure 2.7 shows a comparison between the airspeed derived from the CIP with the airspeed derived from an aircraft pitot-static and temperature sensors. This example is for a CAPS mounted on the C-130 aircraft of the National Center for Atmospheric Research (NCAR) operated by the Research Aviation Facility (RAF).
From this figure we see that the air velocity at the location of the pito-static tube on the CIP is about 15% slower than at the pito-static of the aircraft. The reason for this discrepancy is due to the deceleration of air as it approaches the body of the CAPS. This deceleration causes an increase in the static pressure and a decrease in the dynamic pressure, as shown in the comparison of these parameters with those from the aircraft system, seen in Figs. 2.8 and 2.9. From equation 2.5, we see that these differences are the source of the differences in airspeed.

As explained previously, the user can either select the airspeed derived from the CIP pitot-tube or the airspeed can be derived from an external source, such as from the aircraft system. In the example shown in Figs. 2.7 - 2.9, if the aircraft airspeed had been used to control the sample clock, the particle images would have been slightly elongated since they would have been oversampled by approximately 15%.

![Graph](image-url)
Figure 2.8

CIP Static Pressure vs RAF Static Pressure

CIP \( P_{stat} = 1.0086 \times RA F_{stat} + 14.819 \)

Figure 2.9

CIP Dynamic Pressure vs RAF Dynamic Pressure

CIPQC = 0.76 * RAFQC - 3.3
There have been a number of studies that evaluated the impact of airflow distortion on particle trajectories and orientation (see Appendix I for further reading on airflow distortion and the impact on particle measurements). In general, an airframe presents an obstacle to the airflow that will change the speed and trajectory of particles. For this reason, it is recommended that the velocity measured near to the CIP sample volume be used in preference to a velocity measured in a location farther away, i.e. that from the aircraft system.

### 2.2.2 Out-of-Focus Errors

In section 2.1.2 we discussed the relationship between particle image fidelity and the distance of the particle from the center of focus. Figure 2.10 (from Korolev, 2007) shows a simulation of a cloud drop imaged at various distances, \( Z \), from the center of focus. Remembering that the relationship between DOF and radius is \( \pm 3R^2/\lambda \), for a fixed size, \( R \), and wavelength, \( \lambda \), \( Z_d \) in Fig. 2.10 is the distance from the center of focus, normalized by the optimum distance, \( \pm 3 \).

We see that for \( Z_d = 0 \), the image is completely in focus and the diameter is that of the drop. The three columns on the right side of the figure show how the image could look as recorded by the CIP at 25 μm resolution and depending on the relative position of the drop with respect to the photodiode spacing.

As the drop moves farther from the center of focus, a blank space appears in the center of the image that is a result of diffraction of light around the particle, usually referred to as the Poisson spot. Figure 2.11 shows actual images recorded with a 25 μm resolution CIP-monascale where these ‘donut’ images can be seen.
As described by Korolev (2007), even though these images with the Poisson spot have met the 50% shadowing criterion, they are still larger than the actual particle and corrections are necessary to reconstruct their shape. Details of this procedure can be found in the Korolev (2007) paper, but the technique requires deriving the two parameters, shown in Fig. 2.12, and then looking up the correction factor in a table that is also provided in this same reference.
For example, when the ratios of the image diameter to Poisson spot are 0.1, 0.2 and 0.25, the correction factors are 20%, 30% and 40%, respectively. This correction is one of the selectable option in the PAPI image processing package. Note, however, that particles whose nominal depth of field is larger than the depth of field will not be out-of-focus corrected, i.e. for the CIP with arm width of 100 mm, images larger than about 225 μm will not have a correction applied.

### 2.2.3 Lost Particles

The DMT OAPs, i.e. CIP, MPS and PIP, transmit synchronous and asynchronous information about the particle images. The synchronous, once per second information is a 62 channel size distribution, similar to that described for the light scattering probes. The difference is how the particle sizes are determined. In the OAPs, the size of an image is determined from the maximum number of diodes shadowed during each sample while the particle crosses the array. For spherical particles, this is a reliable measure of the size but may not be the best dimension to measure (see section 2.3) for aspherical images. In addition, images that fall on one or both of the end elements are not included in the frequency histogram. Finally, spurious images, i.e. those produced from shattering (section 2.2.4) will not be rejected may bias the frequency distributions towards larger sizes that will lead to erroneously large LWC or extinction that are derived from the size distributions.

The advantage of having a frequency distribution is that is provides a ‘quick-look’ at the size distribution before a more detailed analysis of the image data (section 2.3). The one-second data is transmitted, regardless of whether there were images recorded, i.e., The number of particle events may often be zero in all 62 channels.

The particle images are stored in separate memory registers, ‘buffers’, and are only transmitted to the data system when filled. The amount of time that it takes to store these registers, that are 64 bits wide by 1024 words long (where one word is equal to 64 bits), depends upon the concentration of particles, their size distribution and the volumetric flow rate. There are two registers of equal capacity to store the images.
When one register is full, the second register is used while the first is transmitted to the data system.

The limitation of this system of altering registers is the rate at which a buffer can be sent to the data system. If the secondary register fills while the primary register is still transmitting its contents to the data system, the instrument will suspend storing particles until it can finish emptying the buffer. During this ‘deadtime’ period, particles that pass through the beam will not have their images recorded; however, their maximum width continues to be monitored and used to update the size histogram that is transmitted each second. Hence, corrections can be applied by comparing the total number of particles recorded per second from the synchronous data with those captured in the image records.

### 2.2.4 Shattering

As in the situation with the light scattering probes, the OAPs have arms that extend from the main body of the instrument that are obstacles to the airflow and that will produce fragments, some of which pass through the laser of the OAP, and can be recorded as particles. Figure 2.13 illustrates some of the typical signatures of such shattered particles. In some cases, all of the fragments produced by the shattering will appear in the same image frame, whereas in other cases these will be a series of particles that typically are very close together in spacing.
In the case of multiple fragments captured in the same image, there are multiple techniques that are used to reject these from the analysis. The most common is to compute the area of a rectangle that circumscribes the maximum width and length of the particle and compare it with the actual number of shadowed pixels. The ratio, $R_f$, will typically be less than ten for non-fragmented particles, whereas values larger than ten are usually a result of many small fragments, with small area, within the same image frame. The value of $R_f$ that should be used to differentiate shattered particles is not a commonly accepted value and should be chosen judiciously after iterating a number of times with a given data set and different values of $R_f$. As discussed in Section 2.3, there is not a universally accepted approach for accepting and analyzing image data and each user has to acquire some familiarity with the techniques.

Shattering fragments that arrive serially are very often spatially distributed with inter-particle distances much shorter than would be predicted from a Poisson probability distribution (see Chapter One, section 1.3). For example, the interarrival time histogram shown in Fig. 2.14 shows that there is a very sharp increase in the probability of arrival times below 30 μs. Above this threshold, the slope of the arrival time distribution is much
flatter as would be expected for the relatively low concentrations of ice crystals. For example, the CIP sweeps out approximately 32 liters per second and the typical cirrus clouds have less than 10 particles per liter. Hence, the average arrival time is 3 ms and the majority of the particles with arrival times shorter than 40 μs are most likely spurious fragments that should be removed. It is interesting to note that there appears to be little sensitivity to the size of the particle, i.e. in the range from 25-150 μm, the probability distributions show approximately the same behavior below 30 μm, showing that when crystals shatter, they can produce larger as well as smaller fragments.

![Figure 2.14](image_url)

Figure 2.14
2.3 Data Analysis

2.3.1 Image Evaluation

Evaluation of individual images measured by the OAPs can range from the very simple to the very complex. Figure 2.15 shows an example image that was captured in a cirrus cloud at -20º C and illustrates some of the more common features that are extracted by automatic image analysis. From this example, one can understand the difficulty in characterizing a particle’s ‘size’ as there are many possible dimensions to use.

Some of the derived diameters that are commonly used, have been published in the literature and are also calculated in the DMT PAPI image analysis routines are:

**Maximum Width** This is the dimension labeled ‘Width’ in Fig. 2.15. The width is the maximum number of diodes shadowed for any slice while the image is passing over the array, where a slice is a single measurement of the ON/OFF state of the photodiodes in the array. This definition of particle size is the same as is used in the once per second size distributions.
**Maximum Length**  This is the dimension labeled ‘Length’ in Fig. 2.15 and is measured by counting the number of slices and multiplying by the resolution of the probe.

**Projected Length**  This is also called ‘Projected Diameter’ (Heymsfield et al., 2002) and is calculated as the hypotenuse of the triangle formed by the maximum width and length, i.e.

\[ D_{\text{proj}} = (W^2 + L^2)^{1/2} \]  

(2.6)

where \( W \) and \( L \) are the maximum width and length, respectively.

**Area**  The area of an image is calculated by multiplying the number of shadowed pixels by the square of the resolution.

**Area ratio**  This is the ratio of the image area to that of the area of a circle circumscribed around the image.

**Perimeter**  The perimeter is estimated by summing all of the transitions from ‘ON’ to ‘OFF’ of the pixels, dividing by \( \sqrt{2} \) and multiplying by the resolution. The perimeter calculated in PAPI includes any gaps or ‘holes’ inside the image.

Figure 2.16 shows how the choice of particle size affects the subsequent size distributions. The histograms are averages over the same time period but with the size defined by the different dimensional criterion. The blue curve labeled ‘1D’ is the size distribution derived from the once per second measurements, the green curve is from the image data, using maximum width as the size of the particle. As expected, the two distributions are almost identical. The only difference is a result of corrections that remove the effects of shattering and correct for out of focus particles. The lavender curve is derived from the area equivalent diameter and the largest difference between its shape and that derived from the maximum width is at sizes larger than 400 μm. The more rapid decrease indicates that ice crystals larger than this size were more irregular in shape so that when collapsed into an equivalent area diameter they decrease in size. The size distribution that uses the projected diameter as its definition for size has higher concentrations at the sizes larger than 200 μm where the other histograms decrease rapidly. The large discrepancy in concentrations is partially a result of the use of the logarithmic scale on the ordinate axis and is also a result of the decreasing effective array width with increasing size that results in a much smaller sample volume for larger particles (see Fig. 2.6).
2.3.2 Derived Parameters

In chapter one, we discuss the usefulness of drawing size distributions in order to examine different features related to concentrations of number, mass extinction, etc. as a function of size. The calculation of the various concentrations is similar when deriving them for OAPs as for the scattering probes, but the varying sample volume with size and the nebulous nature of defining the size modifies somewhat the subsequent derivations. We also introduce here two additional parameters that were not previously discussed, the rainrate and the reflectivity. The formulas given for these parameters can also be used with measurements from the scattering probes; however, these two atmospheric variables are normally dominated by sizes larger than 100 μm except in thin cirrus and marine stratus.
2.3.2.1 Concentration

The total concentration is computed in a similar manner as in (1.3) with the difference that the sample volume changes with size:

\[ C_T = \sum_{i=1}^{m} \frac{n_i}{SV_i} = \sum_{i=1}^{m} c_i \]

(2.7)

where

\( C_T = \) total number concentration (\( \# \text{ cm}^{-3} \))
\( n_i = \) number of particles accumulated in channel \( i \)
\( m = \) total number of size channels
\( c_i = \) concentration per size bin.
\( SV_i = (EAW)(DOF)(V)(T) \)

The EAW is calculated from 2.2 or 2.3, DOF is computed from equation 2.4, limited by the distance between the arms, and \( T \) is the sample time interval. The particle velocity, \( V \), depends on whether the OAP is being used on an aircraft or is employed in a groundbased application. If mounted on an aircraft, \( V \) is either derived from the pitot tube that is part of the CIP, or from some other source such as the DMT AIMM-20 or the aircraft pitot static system.

The DMT meteorological particle sensor (MPS) was specifically developed for the National Weather Service to measure precipitation at the ground. In this application, the particle velocity with respect to the OAP is not constant but is determined by the terminal fall velocity of individual hydrometeors. The MPS measures these directly so that to calculate the concentration of individual size categories,

\[ c_i = \frac{n_i}{EAW_i \cdot DOF_i \cdot V_i} \]

(2.8)

The concentration as defined in (2.7) and 2.8) has units of \( \# \text{ cm}^{-3} \); however, the number concentration of particles larger than 50 \( \mu \text{m} \) is usually much less than one per cubic centimeters. For this reason, we usually express the concentration of these larger particles in units of per liter or per cubic meter by multiplying by \( 10^3 \) or \( 10^6 \), respectively.

Assuming that we make corrections for counting errors due to crystal shattering and missed particles, discussed in sections 2.2.3 and 2.2.4, The estimated uncertainty in number concentration, is \( \pm 20\% \), primarily due to the uncertainty in DOF determination. This uncertainty should actually be expressed as a function of size, since particle’s whose DOF is larger than the distance between the probe arms will have much smaller uncertainty in sample volume than smaller particles. However, given that the number concentration of particles larger than 50 \( \mu \text{m} \) in natural clouds is dominated by particles
between 50 μm and 200 μm, i.e. those whose DOF is smaller than the arm width, the uncertainty is also dominated by errors measuring this size range of particles.

### 2.3.2.2 Liquid Water Content

For spherical water droplets, the liquid water content, \( W_w \), in units of g m\(^{-3} \) is defined as

\[
W_w = \frac{\rho}{6} \sum_{i=1}^{m} c_i d_i^3 = \sum_{i=1}^{m} M_i
\]  

(2.9)

where \( \rho_w \) is the density of water, 1.0\( \times \)10\(^6 \) g m\(^{-3} \), \( c_i \) is in units of # m\(^{-3} \) and \( d_i \) is the diameter of drops in size class, \( I \), in units of meters.

For non-spherical ice particles this equation is not valid since the crystals will not have the density of water and the diameter is no longer a definable quantity. Many studies have been conducted to arrive at a more accurate estimate of the water content of individual ice crystal whose images are recorded by OAPs. Those that are selectable as options in PAPI are the parameterizations that were derived by Heymsfield et al. (2002) and Baker and Lawson (2006). Heymsfield et al. (2002) parameterized the water mass, \( M_i \), for individual crystals, as a function of the projected diameter, \( D_{proj} \), and the area ratio, \( A_r \), defined in section 2.3.1. The relationship has the form

\[
M_i = \sum_{i=1}^{m} \pi D_{i}^{3} k A_{r_i}^{n}
\]  

(2.10)

The choice of the coefficient, \( k \), and the exponent, \( n \), is determined by the assumed habit of the ice crystal. Table 2.1, Table I from Heymsfield et al. (2002), lists these coefficient and exponent values for various crystal habits.
Table 2.1: Coefficients and Exponents Relating Ice Density to Area Ratio as a Function of Crystal Habit  
(Heymsfield et al., 2002)

<table>
<thead>
<tr>
<th>Habit</th>
<th>Habit code</th>
<th>(\rho_c = k(A_c)^n)</th>
<th>(k)</th>
<th>(n)</th>
<th>(r^2)</th>
<th>(A_c)</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columns (theory)</td>
<td>C1a-C1f</td>
<td>0.97(\rho_c)</td>
<td>2.10</td>
<td>1.00</td>
<td>0.05</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>Rosettes (theory)</td>
<td>C2a</td>
<td>1.01 ((\rho_c = 0.81\ g\ cm^{-3}))</td>
<td>2.11</td>
<td>0.99</td>
<td>0.02</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>No. of bullets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.60</td>
<td>1.94</td>
<td>0.99</td>
<td>0.22</td>
<td>0.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.66</td>
<td>2.11</td>
<td>0.99</td>
<td>0.11</td>
<td>0.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.54</td>
<td>2.11</td>
<td>0.99</td>
<td>0.16</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.47</td>
<td>2.16</td>
<td>0.99</td>
<td>0.20</td>
<td>0.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.49</td>
<td>2.16</td>
<td>0.99</td>
<td>0.22</td>
<td>0.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.49</td>
<td>2.25</td>
<td>0.99</td>
<td>0.26</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.50</td>
<td>2.35</td>
<td>1.00</td>
<td>0.29</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.52</td>
<td>2.52</td>
<td>0.99</td>
<td>0.32</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Side planes, S1–S3, (Observatory)</td>
<td>0.35</td>
<td>2.34</td>
<td>0.79</td>
<td>0.18</td>
<td>0.98</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Planar crystals               |            |                        |        |        |         |         |        |
| Wind tunnel                   | Pla-Plb-Plc | 0.084                  | 2.38   | 0.97   | <0.79   |        |        |
| (Takahashi et al. 1991)       |            |                        |        |        |         |         |        |
| Sfc. observations            | Many types | 11.96                  | 22.84  | 0.48   | 0.79    |        |        |
| (Heymsfield and Kajikawa 1987) |            |                        |        |        |         |         |        |
| Aggregate \(A_c = 0.83\)     |            |                        |        |        |         |         |        |
| Aggregate \(\rho_c\) vs D relationships \(\rho_c = \chi D^\eta\) | | | | | | | |
| Aggregate study               | Component crystals | \(\chi\) | \(\eta\) | | | | |
| Magone and Nakamura (1965)    | Planar crystals | 0.010                  | 1.50   | 0.28   | 1.0     |        |        |
| Kajikawa (1982)               | 2–5 planar crystals | 0.015              | 1.50   | 0.13   | 0.77    |        |        |
| Agg. side planes (Sfc.)       | S1–S3      | 0.18                   | 1.52   | 0.97   | 0.21    | 0.85    |        |
| CPI observations (ARM)        | Rosettes   | 0.16                   | 1.48   | 0.99   | 0.16    | 0.56    |        |
| Aggregate hybrid approach     | C2a        | 0.035                  | 0.96   |        |         |        |        |

As can be seen in Table 2.1, the values for \(k\) and \(n\) vary widely depending on the shape of the ice crystal, i.e. column, plate, aggregate, etc. The challenge is to determine a particle’s shape on a particle by particle basis. A number of automatic pattern recognition techniques have been developed to do this type of classification, some of which are, or will be, incorporated in the PAPI image processing routines. A description of these algorithms is beyond the scope of this manual but details can be found in Durore (1982), Durore, et. al. (1994), Heymsfield and (1978a,b; 1987), Holroyd (1987), Hunter et al. (1984), Korolev and Sussman (2000), Rahman et al. (1981)

An alternative to using the technique of Heymsfield et al. (2002) is implementation of the algorithm derived by Baker and Lawson (2006). They used a data set of ice crystals were captured on slides, photographed then melted to get the water content of individual crystals (Mitchell et al., 1990). After classifying the crystals into various habits, and
measuring the areas, perimeters, and characteristic lengths, using the photographs, they determined that the relationship with the best correlation (> 0.8) from which ice water mass could be determined was with the area, A, of the image:

\[ M_i = 0.115A^{1.218} \quad \text{(units are in mg)} \]  

(2.11)

This definition of ice water mass is also incorporated in the image analysis routines implemented in PAPI.

The uncertainty in liquid or ice water content determination is related to the errors in measuring the particle dimensions, estimating the sample volume and deriving a representative density. Formulations such as (2.10) and (2.11) were developed to decrease the uncertainties but are still sensitive to errors in \(D_{proj}\), \(A_r\), and \(A\). Various estimates have been made of the water mass uncertainty derived from OAP images, but there is not a general consensus as to the magnitude of these uncertainties. The best case uncertainty is approximately ±40% but likely exceeds ±100% in the majority of cases.

### 2.3.2.3 Rain Rate

The rain rate is defined as the height of a column of water that forms over a unit area during a given period of time. The convention is to use dimensions of mm hr\(^{-1}\). When a size spectra of precipitation particles is being measured, the rain rate, \(RR\), is calculated as

\[ RR = T \sum_{i=1}^{N} \frac{V_i w_i}{\rho_i} \]  

(2.12)

where \(V_i\), \(w_i\), and \(\rho_i\) are the particle fall velocity, liquid water content, and density in size category, \(i\), of \(N\) categories. The units for \(V\), \(w\) and \(\rho\) are mm hr\(^{-1}\), g m\(^{-3}\) and g m\(^{-3}\), respectively.

The uncertainty in (2.12), is estimated from the propagated errors in the derived water content, particle density and fall velocity, using the root-sum-square method described in Chapter one, section 1.3.2. The least error when calculating the RR is in all water precipitation when we know the density of the particles and can measure their size, after depth of field corrections, to ±20%. Taking into account the uncertainties in sample volume, we estimate the uncertainty in the liquid water content as ±40%.

If the particles fall in still air, the fall velocity is the drop’s terminal velocity (see next section), a quantity that has been well defined by laboratory measurements (Gunn and Kinser, 1949; Beard, 1977) with less than 10% uncertainty. Turbulent air will add an
extra vertical motion component, either positive or negative, to the drop’s fall velocity. This is a source of uncertainty if the vertical wind or drop velocity is not directly measured as it is in the MPS. If the particle velocity is not measured directly, the estimated uncertainty in this variable is ±20%, otherwise, measuring it directly lowers this uncertainty to ±10%. Propagating the uncertainties in velocity and water content, the expected error in estimating water content is ±40% (MPS). to ±45% (CIP).

Additional uncertainties arise when precipitation hydrometeors are not liquid, e.g. snow, graupel, sleet or hail. In these cases the particles can be aspherical with variable density. The drag force is sensitive to particle shape and the gravitational force depends on density. There have been many studies that relate the fall velocity to particle dimension (see section 2.3.2.4) but all these parameterizations require a priori information about the habit of the ice crystals. For this reason the terminal velocity is difficult to predict and uncertainties will exceed ±50%.

As summarized in section 2.3.2.2, the water content of ice particles is highly variable and the derived values may be in error by more than 100%. Hence, with no a priori knowledge of the particle characteristics, estimated rain rate uncertainties can readily exceed 100-150%.

### 2.3.2.4 Fall Velocity

The fall velocities of drizzle and precipitation particles depend upon their terminal velocities and the vertical air velocity. The terminal velocity is the steady state velocity attained by an object when the gravitational force is balanced by the air drag force. For water drops, this velocity has been measured in the laboratory and analytical expressions have been derived that accurately predict the relationship between terminal velocity, drop diameter, ambient temperature and pressure. Figure 2.17 shows the relationship between fall velocity and droplet diameter as measured by Gunn and Kinser (1949) and Beard (1977). Also shown is the least squares fit to the Gunn and Kinser data:

\[
V_t = -19.27 + 0.50D - 9.04 \times 10^{-5} D^2 + 5.66 \times 10^{-9} D^3
\] 2.13

The terminal velocities of other types of precipitation particles, such as hail, graupel, snow pellets, sleet, or snowflakes, are much less predictable since neither the gravitational or drag force can be well defined, due to the diverse masses and shapes that these particles have. Many studies have been conducted relating ice particle terminal velocity to length, width or mass (see Mitchell, 1996 for a summary of these studies), but a priori knowledge of the crystal habit is essential to apply any of these relationships with confidence.

When making groundbased measurements with the MPS, we determine the fall velocity of each particle directly by measuring the particle residence time, \(T\), over the diode array.
The width of the particle, derived from the maximum number of diodes shadowed across the array, provides an estimate of the particle size, $D$. The particle velocity, $V_t$ is then $D/T$.

The residence time is measured by counting the number of cycles of a 2 MHz clock that occur while the particle shadow is on the array. The digitization uncertainty is approximately $\pm 25 \, \mu m$ (one-half the probe resolution) and the residence time accuracy is $\pm 0.25 \, \mu s$. From this we can estimate the uncertainty as a function of particle size and terminal velocity using propagation of errors. The error in estimating terminal velocity decreases rapidly as particle size increases because the digitization error dominates the terminal velocity error and decreases with increasing size.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{terminal_velocity_graph}
\caption{Terminal Velocity vs. Droplet Diameter}
\end{figure}
2.3.2.5 Reflectivity

The reflectivity, Z, is a parameter that is used to relate the amount of microwave energy returned to a radar from an ensemble of particles and is defined as

\[ Z = \sum_{i=1}^{N} c_i D_i^6 \quad \text{(units of mm}^6 \text{ m}^3) \]  

2.14

One of the powerful uses of radar is to estimate the rainfall amounts from the direct measurement of reflectivity. OAP measurements are one of the mechanisms for deriving a relationship between rainrate and radar reflectivity. As shown in the example of Fig. 2.18, a power-law relationship of the form \( Z = aR^b \) describes the relationship very well.

This figure, that uses the ground-based measurements from the MPS, also illustrates two other very important features. First of all, as discussed previously, the rainrate can be calculated from the droplet spectra and terminal velocities derived from laboratory results (Gunn and Kinser, 1949; Beard, 1977) or measured directly. The colored, filled circles, and the least squares fits, show how the resulting rainrates differ depending which particle velocity is used. The exponent of the Z-R relationship is identical but the coefficients differ by a factor of two. This implies that the rainrate, if predicted from the reflectivity, would be underestimated by a factor of two if the Z-R relationship using the Gunn and Kinser terminal velocities instead of those directly measured.

The second feature drawn on the graph is a Z-R power-law relationship that is often assumed by radar meteorologist when estimating rainfall from reflectivity. The coefficient and exponent are quite different and, in comparison with the Z-R relationship derived from the MPS using the measured terminal velocities, the derived rainfall rate would be substantially smaller.

The uncertainty in Z is quite large because of the sixth power relationship to particle diameter. Ignoring at the moment the considerable error in determining D for non-spherical particles, for uncertainties in number concentration, c, and diameter, D, of 20% for water droplets, the subsequent, root-sum-squared error in determining Z is approximately ±53%. This error increased to more than 200% when the uncertainty in diameter is larger than 20%. In addition, for non-spherical particles, the equation for Z is no longer applicable and needs to be replaced with a more complex formulation that takes into account particle shape and the associated change in dielectric constant.
Figure 2.18

Rain Rate (mm/hr)

Reflectivity (mm$^6$/m$^{-3}$)

10 100 1000 10000

1E+003 1E+004 1E+005 1E+006 1E+007

With Measured Fall Velocity

$Z = 651 RR^{1.03}$

With Gunn and Zinzer Fall Velocities

$Z = 1324 RR^{1.04}$

$Z = 200 RR^{1.5}$ (Rogers, 1979)

Form DOC-0223

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2.4 Further Reading on Single Particle Imaging


Heymsfield, A.J. and J.L. Parrish, 1978b: A computational technique for increasing the effective sampling volume of the PMS Two-Dimensional Particle Size Spectrometer, J. Applied Meteor., 17, 1566-1572.


